CORRELATION VERSUS CURVE FITTING IN RESEARCH ON ACCIDENT PRONENESS: REPLY TO MARITZ

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Maritz (3) states that the technique of correlating accident records in two successive periods is indispensable as evidence of accident proneness. He suggests that the fitting of theoretical Poisson and Negative Binomial distributions is not an adequate criterion of the absence or presence of differences in accident proneness in a group. There are certain weaknesses in this position, and a clarification is demanded. To reiterate the major points of our earlier paper (4):

1. Personal accident proneness, a component of accident liability, has been overemphasized.
2. This can be demonstrated by a method which reveals the extent to which accident records *could* be attributed to differences in accident liability, and this was found to be 20 per cent to 40 per cent of the total variance of accident records.
3. This method is based on the use of univariate distributions.

We agree with Maritz that a good Negative Binomial fit does not prove the existence of differences in accident proneness with mathematical certainty. We never said that it did. It is doubtful whether anything is ever proved with mathematical certainty in empirical sciences.

The issue raised by Maritz is that the correlation technique is indispensable in the establishment of accident proneness and that the evidence from a univariate distribution is invalid. We strongly disagree. We shall show that the two techniques give much the same information and, therefore, either can be offered as evidence for accident proneness. In fact, neither is wholly conclusive. Detailed histories of individuals' accident careers should be better than either.

Maritz views correlations as the "direct technique" of establishing differences in accident proneness. His claim is that "the most direct method of establishing proneness in a group of people all of whom ought to be exposed to the same environmental risk, consists of splitting a lengthy period of observation into two periods and correlating the frequency of accidents per individual for these two periods. This statistical technique is nearest to the psychological definition of accident proneness." He attempts to show that this technique may contradict conclusions based on univariate distributions. His evidence is based upon two hypothetical distributions and a previously unpublished

distribution by Adelstein. One of his hypothetical distributions resembles a chance pattern and yet yields a correlation in successive periods. The other is suggestive of differences in accident proneness but is uncorrelated in successive periods. Adelstein's data, in his opinion, illustrate two simple chance distributions which are correlated with each other.

Maritz' two hypothetical distributions illustrate the mathematical possibility for two Poisson distributions to be correlated, and for a Negative Binomial Distribution to result from summation of two uncorrelated distributions. However, the occurrence of both kinds of bivariate distributions in the case of accident records is unlikely. The mathematical derivation of the bivariate correlated Poisson distribution which Maritz applies to accident records shows only that it can be approximated by drawing colored balls from enclosed boxes. We do not believe that such a distribution should be expected in the case of accident records. The only obvious meaning of a Poisson distribution of accident records is that it is satisfactorily explained by the assumption of equal and constant accident liability. Without such an assumption it looks like a result of an odd coincidence. If Poisson distributions of accidents are the results of equal liability, they should be uncorrelated, and it is not clear what kind of combination of circumstances should lead to the expectation of Poisson distributions correlated in successive periods.

In his other hypothetical distribution this lack of plausibility is obvious. For example, all six hypothetical individuals who had more than eighteen accidents each in one period had zero accidents in the other period. This could occur, but is hardly to be expected. In other words, in this example, Maritz proves that it is possible to obtain a correlation of $-.11$ by arranging numbers in a manner designed to obtain it. This is granted, but what does it prove about accidents?

The only empirical material Maritz presents is the unpublished Adelstein data, and it must be regarded with more seriousness than his two hypothetical distributions, which are mathematically possible but highly improbable. Maritz claims that the examination of the univariate distributions suggests a "pure chance pattern," but that the correlation between the accident records in the two periods reveals the existence of differences in accident proneness to the extent of a correlation of .29. The existence of the correlation is treated as something that could not have been foreseen in terms of the accident distributions in the two observation periods. Thus, the impression is created that the combined period had properties which were essentially different from

those of the shorter periods when considered alone. He bases his interpretation on the fact that three χ^2 tests fail to reveal significant differences either between two Poisson distributions and Adelstein's five- and six-year distributions, or between a bivariate correlated Poisson distribution and Adelstein's scatter diagram for the two periods.

In dealing with these data Maritz makes the common error of confusing the failure to disprove a hypothesis with its proof. He states: "Equation [1] was fitted to the observed data of Table III and the resulting test of goodness of fit gave for 7 *df*, $P=.49$. Hence it follows that the above data follow a *correlated* bivariate Poisson distribution" (p. 438). This second sentence does not follow from the first. The failure to disprove a hypothesis according to which the data for a combined period have properties (the .29 correlation) different from those of the constituent periods (viewed as exhibiting a simple chance pattern) is not the same as proof of it. A closer examination of the data for direct evidence of this type of heterogeneity fails to reveal anything convincing, and the opposite and theoretically more plausible hypothesis of essential homogeneity of the eleven-year observation period fits the data more closely than Maritz' heterogeneity (i.e., bivariate correlated Poisson) hypothesis.

In terms of the properties of the accident distributions in the two consecutive periods, the most probable correlation between the accident records in these periods is not zero as Maritz implies, but .21. This is quite close to the observed .29. The estimate of a correlation of .21 was arrived at by determining the estimated percentages of the accident records attributable to factors other than chance (18 per cent and 24.4 per cent) and then computing their geometric mean. In determining these percentages the formula $(v-m)/v$ was used; the two variances were 1.485 and 1.370, respectively, and the two means were both 1.123.

If the eleven-year period had no properties essentially different from those of the shorter period, it should be possible to construct theoretical scatter diagrams approximating the empirical one by utilizing statistics derived either from the two periods considered separately (without considering their correlation), or from either one of them taken alone, or from the total period taken as a whole. The form of the bivariate distribution chosen was the bivariate Negative Binomial. It is based on much the same assumptions as those made by Greenwood and Yule (1) in their derivation of a univariate unequal liability distribution, namely:

Accident liability is distributed in people in accordance with a Pearson III curve.

Accident liability of a person remains constant per unit of time throughout the two observation periods.

Each particular degree of accident liability gives a simple chance (Poisson) distribution of accident records in each observation period (p. 279).

Table I presents the Adelstein data together with a theoretical distribution constructed in accordance with these assumptions; the computation utilizes only the mean and the variance of the first period, and the fact that the second period lasted six years while the first one lasted five years. The formula for the theoretical frequency for the cell representing j accidents in the first, k accidents in the second is

$$y_{jk} = Na^k\left(\frac{c}{c+a+1}\right)^p \frac{\Gamma(p+j+k)}{\Gamma(p)j!k!(c+a+1)^{j+k}}$$

in which a is the ratio of the two durations, and c and p are two constants derived from the mean and variance of the first period, as follows: $c=m/(v-m)$, $p=m^2/(v-m)$. (The derivation of the formula, which closely resembles that of Greenwood and Yule, will be published elsewhere.)

TABLE I

A COMPARISON OF ADELSTEIN'S ACTUAL ACCIDENT DATA WITH THE THEORETICAL BIVARIATE DISTRIBUTION COMPUTED FROM HIS FIRST-PERIOD DATA

($m=1.123$, $v=1.370$)

First Period	*Second Period*								
	0	1	2	3	4	5	6	7	Total
0	21(16.3)	14(14.8)	8(8.0)	1(3.3)	—(1.2)	—(0.4)	—(0.1)		44(44.1)
1	17(12.3)	12(13.4)	8(8.4)	3(4.1)	1(1.6)	—(0.6)	—(0.2)	1(0.1)	42(40.7)
2	6(5.6)	9(7.0)	2(5.1)	2(2.7)	2(1.2)	—(0.5)	—(0.2)	—(0.1)	21(22.4)
3	1(1.9)	1(2.8)	3(2.3)	3(1.4)	1(0.7)	—(0.3)	—(0.1)		9(9.5)
4	1(0.6)	3(1.0)	—(0.9)	—(0.6)	—(0.3)	—(0.1)			4(3.5)
5	—(0.2)	—(0.3)	—(0.3)	2(0.2)	—(0.1)				2(1.1)
6	—(0.1)	—(0.1)	—(0.1)	—(0.1)					0(0.4)
Totals	46(37.0)	39(39.4)	21(25.1)	11(12.4)	4(5.1)	0(1.9)	0(0.6)	1(0.2)	122(121.7)

The theoretical distribution appears to fit the data quite well. The χ^2 computed was 8.743, $df=10$, $P=.58$. It follows that the correlation technique recommended by Maritz did not add anything significantly new to the information one could gather by examining one of Adelstein's univariate distributions.

In arguing for correlations and against properties of univariate distributions as grounds for assuming differences in accident proneness,

Maritz overlooks the fact that these two kinds of statistical measures are closely interrelated. The correlation between accident records in two periods can always be computed from the variances in these two periods and in the total period, according to the formula

$$r = \frac{V_{1+2} - V_1 - V_2}{2\sqrt{V_1 V_2}}.$$

Similarly, the increase in accident variance when one combines two observation periods is an increasing function of the correlation between these periods, in accordance with the elementary formula $V_{1+2} = V_1 + V_2 + 2r\sqrt{V_1 V_2}$. Inasmuch as this formula is exact, and inasmuch as every observation period has an early stage when the variance is smaller than the mean, a distribution cannot have a variance greater than the mean value unless there were positive correlations between successive periods somewhere in the past. Similarly, a Poisson distribution cannot result unless the accident records in the subdivisions of the observation period were uncorrelated or unless the effects of positive and negative correlations cancelled each other. If an examination of two univariate distributions in two successive periods suggests something markedly different about the existence of accident proneness, compared to the correlation between these periods, the interperiod correlations must have similarly changed in the past. Maritz' hypothetical distributions could be used just as readily in arguing against the use of correlations in accident research as against the use of variances. The only advantage of a correlation lies in the fact that it enables one to tell a particular time when the variance of a set of accident records has risen at a rate beyond chance expectation.

A further consideration with reference to the correlational technique is that it presents certain practical difficulties. Accident proneness is a problem for industry as well as for the theoretical statistician. From the point of view of industry it is often difficult to acquire data on the same individual for two successive periods. Those with high accident rates in the first period are likely not to be found in the second period. They may be dismissed or resign from their jobs, not to mention being hospitalized or dead. The study reported by Kerr (2) is typical of practical problems confronting industry. The major effort is to reduce accidents, not to wait for successive periods.

Summary

1. The hypothetical distributions presented by Maritz are mathematically possible and demonstrate the lack of mathematical certainty

of inference from empirical data, but such distributions are not likely to be encountered in practice.

2. Correlational research on accident proneness is legitimate, but inferences about accident proneness drawn from correlations are not more certain than inferences drawn from the fitting of distributions.

3. Correlational research is not always feasible for practical reasons. In any event, it is not indispensable with reference to establishing accident proneness.

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